

**Exercise 21**

Given  $f(x) = \sqrt{2-4x}$  and  $g(x) = -\frac{3}{x}$ , find the following:

- (a)  $(g \circ f)(x)$
  - (b) the domain of  $(g \circ f)(x)$  in interval notation
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**Solution**

Compute  $(g \circ f)(x)$  by plugging the formula for  $f(x)$  where  $x$  is in the formula for  $g(x)$ .

$$\begin{aligned}(g \circ f)(x) &= g(f(x)) \\ &= -\frac{3}{(\sqrt{2-4x})} \\ &= -\frac{3}{\sqrt{2-4x}}\end{aligned}$$

It's impossible to divide by zero and to take the square root of a negative number, so

$$\sqrt{2-4x} \neq 0 \quad \text{and} \quad 2-4x \geq 0.$$

Square both sides of the equation on the left.

$$2-4x \neq 0 \quad \text{and} \quad 2-4x \geq 0$$

Combine the two conditions.

$$2-4x > 0$$

Solve for  $x$ .

$$-4x > -2$$

Divide both sides by  $-4$ .

$$x < \frac{1}{2}$$

Therefore, the domain of  $(g \circ f)(x)$  in interval notation is  $(-\infty, \frac{1}{2})$ .